

Calculation Methods

Key Stage 1 - Key Stage 2

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Introduction

The purpose of this booklet is to provide guidance and information about the types of calculation methods that children are being taught, and are using up to the end of Key Stage 2 (Year 6 children are at the end of Key Stage 2).

Much has changed in the teaching and learning of maths over the past few years, not least with the introduction of the "National Numeracy Strategy". This strategy provides a "framework" which is used by schools and individual teachers to plan and teach maths lessons from Reception Year to Year 6. The "Key Stage 3 Strategy" provides a similar structure for Years 7 to 9. The calculation methods used by children today are in many cases different from those used by adults when they were at school. This can cause anxiety, with parents and carers unsure whether or not they should teach children particular methods.

As a general rule, if your child brings home some maths work which involves calculations:

- Ask them to explain how they would solve this at school, and to explain to you the methods they have been taught. Use this booklet to help.
- If your child is unable to explain their method, or unsure about what to do, the best advice is to contact your child's teacher.

Parents and carers naturally want to help their children, and schools welcome this support. We all have the children's best interests at heart and we all need to be pulling in the same direction!

The calculation methods taught today gradually build on the children's understanding over a period of time. They have been introduced after research programmes have shown them to be effective. The aim is to teach children calculation methods which they understand, can use correctly, and can use confidently to solve problems.

Calculation methods

Mental and written calculations.

When most of today's adults were at school there were basically two ways of working something out without a calculator:

Mental calculation- which you did in your head, or not at all.

Written calculation- which usually involved lining numbers up one above the other, like this:

$$\begin{array}{r} 36 \\ + 57 \\ \hline 93 \\ 1 \end{array}$$

Methods like this are called "column" methods because they involve lining the digits up in columns.

The National Numeracy Strategy gives a great deal of emphasis to children learning to use a whole range of mental calculation methods properly, **before** they move on to written calculations. These mental methods will involve the children writing or drawing things to help them. These are often called "jottings" and will not look like the example above, with the numbers lined up. They might well involve using a number line, a method which we will look at later.

This does not mean that written methods are not seen as important. It is expected that children in Year 6 will have a written method for each operation $+$ $-$ \times \div which they can use reliably to solve problems. The written methods that children use will not necessarily involve lining the numbers up in columns, since there are other effective methods which we will look at in this booklet.

Using a calculator

The correct use of a calculator is an important part of children's mathematical understanding. The National Numeracy Strategy sets out a range of calculator skills to be taught in years 5 and 6 and these include skills such as: Using the [clear] and [clear entry] keys properly, understanding the display when solving money and measurement problems, using the memory correctly, and using the calculator to find decimal equivalents of fractions. For example the calculator display might say 23.4 but if we are solving a money problem this actually means £23.40.

An important calculator skill is the ability to estimate, and have a feel for the "size" of the answer so that you know if you have made a keying error.

These are quite complicated skills, and children need plenty of practice with a calculator before year 5. Children will not be using the calculator to solve simple calculations that can be worked out using other methods, but they might use the calculator in any year group for exploring numbers, problem solving and investigating patterns, and this also provides practice at using the calculator keys correctly.

In summary:

- Children are taught a whole range of mental calculation methods which they will use to solve problems.
- Children will be taught to write things down to help their mental calculations, such as number lines.
- When children use written methods for calculation, these methods will not necessarily involve lining numbers up in columns, but might use number lines or grids.
- The use of the calculator should not be seen as "cheating". To be able to use a calculator correctly to solve problems is an important part of children's mathematical development.

Mental methods for addition and subtraction

As we have seen earlier, children are taught a whole range of methods for calculating mentally. For addition and subtraction these include:

Knowing all the pairs of numbers which add up to 10, eg. $8 + 2 = 10$
 $6 + 4 = 10$ and so on. Children need to know these facts instantly, and later learn and use the numbers that add up to 20, eg. $17 + 3 = 20$
 $11 + 9 = 20$ and so on.

Knowing the subtraction facts which correspond to these additions, eg. $10 - 8 = 2$, $20 - 17 = 3$ and so on.

Knowing that when we are adding, we can add the numbers in any order (but that this does not work for subtraction!) and that it helps to put the larger number first, eg. $4 + 17$ is easier if we put the 17 first then add the 4.

Children learn the "doubles" of numbers, and use this knowledge to work out examples such as $6 + 6 = 12$, and extend this to examples such as $6 + 7 = \square$. We know our doubles, that $6 + 6 = 12$ so $6 + 7$ must be 1 more which is 13.

Examples such as $23 + 19$ can be solved easily by adding 20 then taking 1 off: $23 + 20 = 43$, one less is 42

Children are taught to understand how our number system works in that the number 23 means "two tens and three ones" and that the 4 in 45 stands for "four tens" which is 40. As children develop, this understanding is extended to hundreds, thousands etc. and into decimals.

These are a few examples of the many strategies that children are taught to use when adding and subtracting. Children are encouraged to explain how they work things out, compare their methods with those of other children and the teacher, and see how different methods for calculation are suited to particular examples.

The Empty Number Line

We have discussed earlier how children are taught to write things down to help with their mental calculations, and that one example of this is to use a number line. We will now look at how an empty number line can help with mental calculation.

This is an empty number line. It is extremely useful and important for developing children's mathematics.



The line has no markings or scale, it does not need to be drawn neatly with a ruler. It does not matter which way up it is; it can be vertical, horizontal or anything in between and it does not even need to be straight. The power of the empty number line lies in its ability to provide an image or picture of a calculation which can develop children's thinking about the structure of numbers, and move on from simply counting on in "ones".

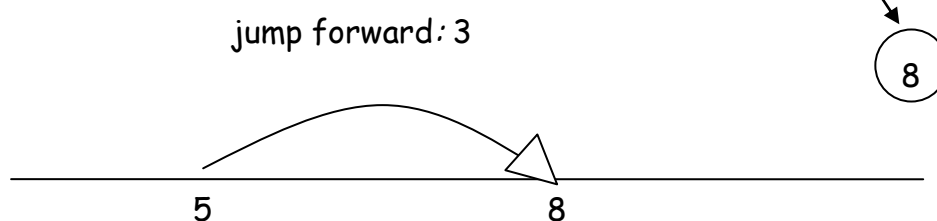
Once children are used to using the empty number line, this can help to improve their mental calculations by providing a picture in their minds (a mental image) so although they might not need to draw a line to help them, they are using one mentally.

The empty number line is flexible in that it can be drawn quickly and easily anywhere, and can be used to represent an infinite range of numbers, including decimals. It is an excellent way for children to record their mathematical thinking, it can be used effectively by young children who are just beginning to record their mathematics, through to older pupils requiring a calculation method for subtraction involving decimals.

Simple examples for addition using the empty number line.

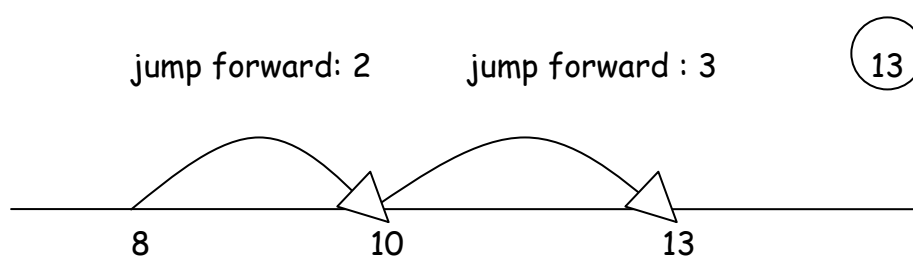
One convention that can be useful when using an empty number line is to circle the answer to the calculation to the right of the line. This encourages children to make links between the use of the line, and more formal systems for recording calculations, as well as making the answer stand out clearly.

This line represents $5+3=8$ The answer, 8 has been circled.



This addition: $8+5=$ can be solved using the empty number line to jump up to the next ten to make the calculation easier.

"Tens numbers" - numbers ending in zero- always make good landmarks, or "comfortable resting places" to make the calculation easier:



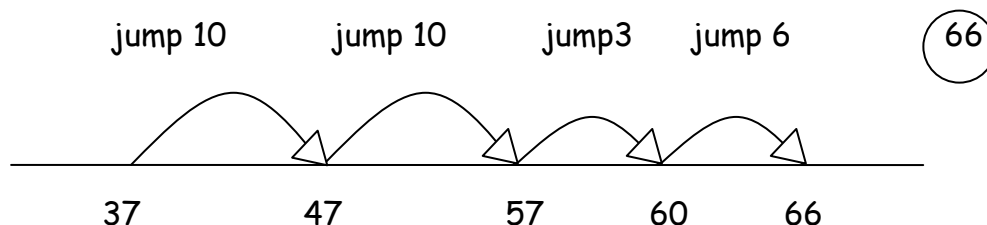
We use 2 of the 5 to jump up to 10, we know that there are 3 left and jump on to 13. The empty number line helps children to develop their mental calculation, and move on from simply counting on in ones.

Methods for addition using the empty number line

There are three mental methods for addition for which the empty number line is useful. These three methods can also be used for subtraction, but in this section we will look at each method in turn for addition, using the same calculation, $37 + 29 = 66$. The methods have been given names so that we can refer to them easily.

The "Tens jumping" method

This first method is the simplest mental image and simply involves jumping along the line in tens, from our starting number.

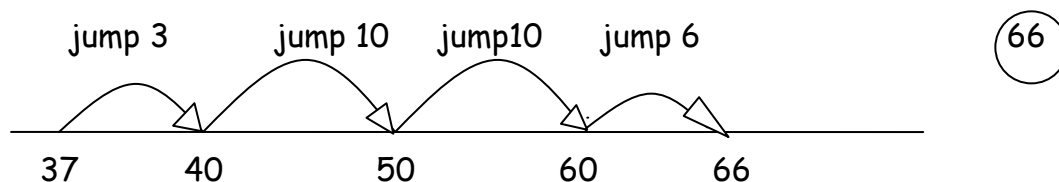


We keep the 37 whole, then jump in tens to 47, then 57 to add the 20, then jump to 60 to help cross the tens. Having used 3 of the 9 to jump from 57 to 60, we then have a final jump of 6 to reach 66.

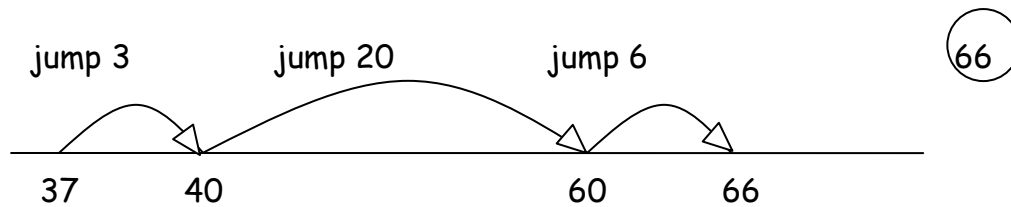
The "Hit the tens" method

This method involves a first jump to the next "tens number" (multiple of ten), to find a "comfortable resting place", making the rest of our jumps much simpler.

Once again we will solve $37 + 29 = 66$



Very often when children use this method they are more able to perform a single jump of several multiples of ten from a "tens number" or "comfortable resting place":

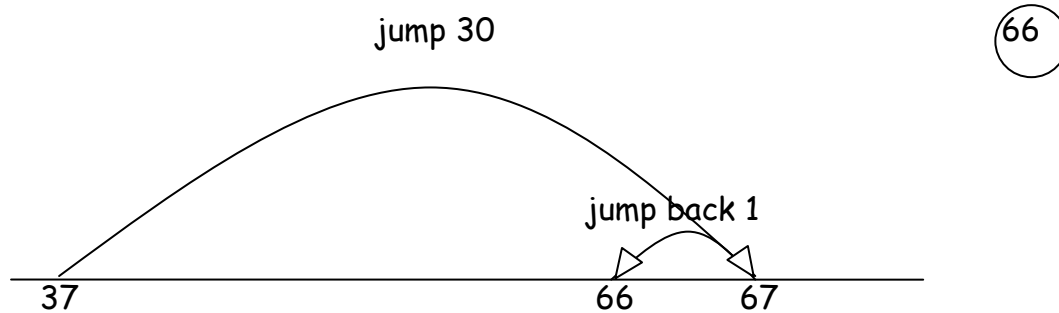


As in this example, we have made a single, quite easy jump of 20 from 40 (a "comfortable resting place") to 60. This makes the calculation more efficient, with one less step. We are then left with a final jump of 6.

The "Overjumping" method

As before we will solve $37 + 29 = 66$

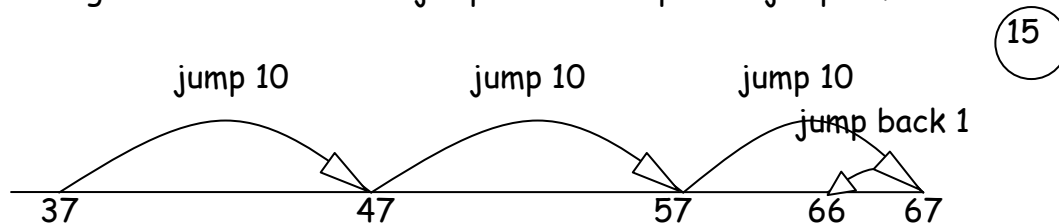
For this method we use the fact that it is very easy to add a "tens number" (multiple of 10), so we round up our second number to the next "tens number" make an "overjump", then jump backwards. In this case we make a first jump of 30, and jump back 1, because we are actually adding 29.



With this method we are performing $37 + 29 = 66$
as $37 + 30 - 1 = 66$

This "overjumping" method is particularly suited to adding numbers like 29 which are close to the next "tens" number.

Some children might not be confident about making a single jump of 30, so might need to make the jump in three separate jumps of ten.



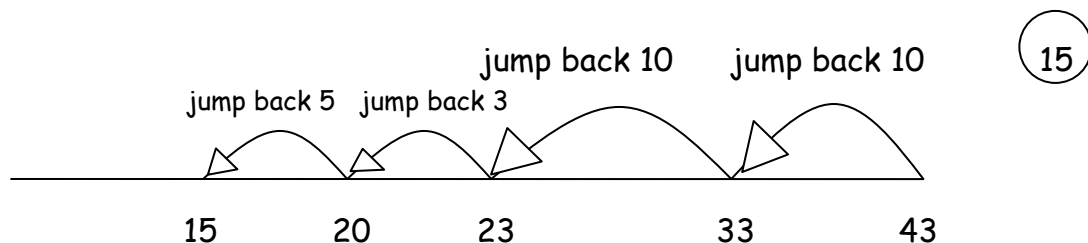
Methods for subtraction using the empty number line

We will now look again at each of the three empty number line methods, but this time for subtraction. We will use the calculation $43 - 28 = 15$ as an example.

The "Tens jumping" method

$$43 - 28 = 15$$

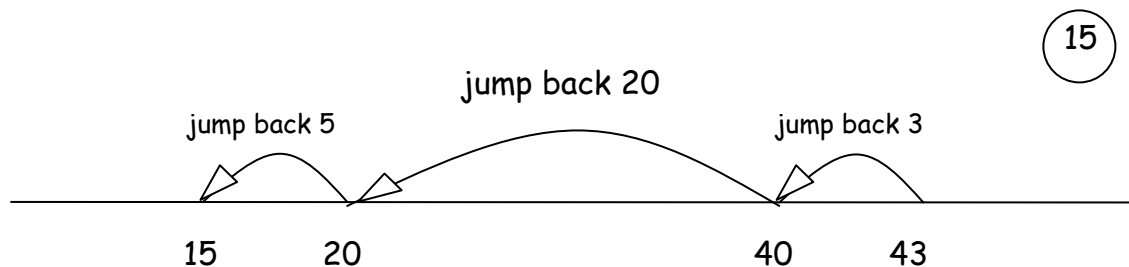
This again is the simplest method and can be thought of as "taking away" ten at a time by jumping backwards, first in tens to 23, then back 3 to 20 and a final jump back of 5 to 15.



The "Hit the tens" method

$$43 - 28 = 15$$

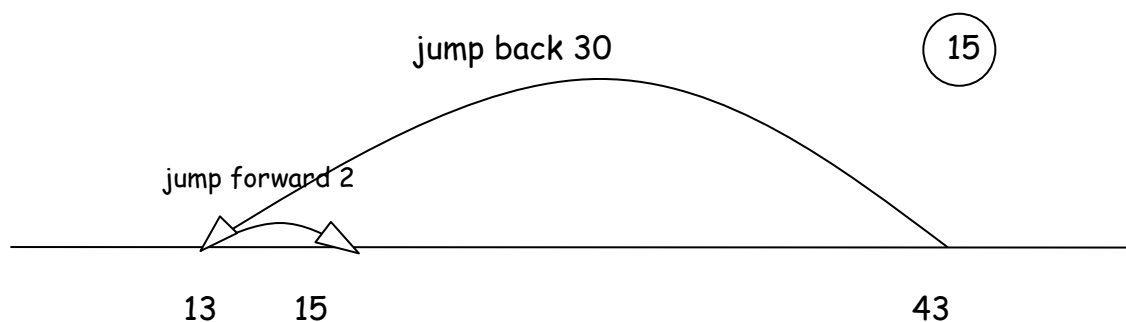
This method involves an initial jump back of 3 to 40, to hit the first "tens number" (multiple of ten) below 43. Once on a "tens number", we have a "comfortable resting place" which can enable single jumps of several multiples of ten, in this case a single jump back of 20. We then have a final jump back of 5



The "Overjumping" method

$$43 - 28 = 15$$

This method involves jumping back too far, in this case making a jump back of 30 to 13, then making up for this by jumping forward 2 to 15. This example shows a single backward "overjump" of 30, although this could be performed by some children as three backward jumps of 10.



We have performed $43 - 28 = 15$
as $43 - 30 + 2 = 15$

This method can be thought of as "take too much, add back" and is sometimes called "compensation"

Subtraction by "counting on"

The three empty number line methods we have looked at so far are similar for addition and subtraction. There is another method for subtraction only, which involves counting on from one number to another. This method can be used with an empty number line, and can also develop into a "column" method for subtraction which we will look at later in the **Written methods for subtraction** section.

If children are to use this method well, they need to understand that subtraction does not only mean "taking away", but that it can also mean "difference".

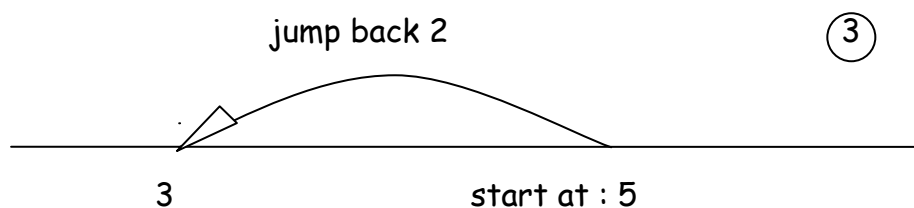
Subtraction as taking away and as difference

Children first learn about subtraction as "taking away", which is how we have considered subtraction in our examples so far. Let us now think of a simple example:

$$5-2=3$$

If I have 5 counters and take 2 away, how many are left? Three!

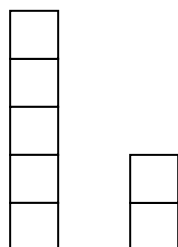
Using the empty number line :



But subtraction can mean other things as well as taking away. Children need to understand that $5-2$ can also mean:

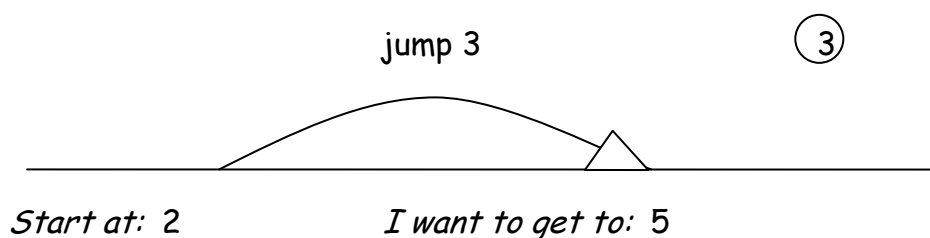
What is the difference between 5 and 2?

This can be illustrated by making a tower of 5 cubes, and a tower of 2 cubes.



When we compare the two towers, there is a difference of 3 cubes in their height.

We can illustrate this on the empty number line by thinking of $5-2$ as: *what must I add to 2 to get to 5? Or: If I start at 2, how many do I need to count on to get to 5?*



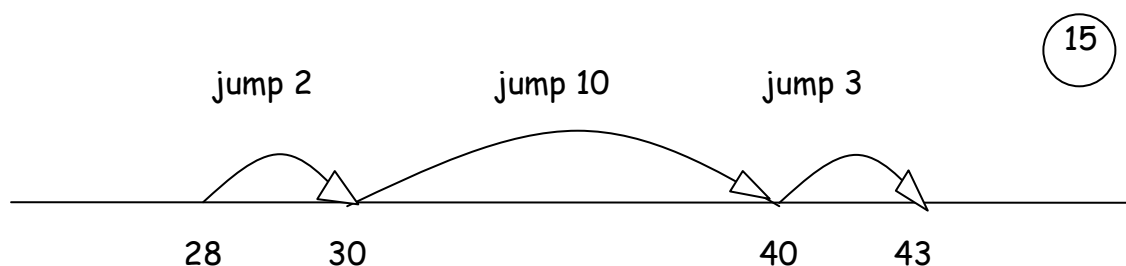
We have solved the simple subtraction $5 - 2 = 3$ by "counting on" from the two to the five. This time our answer is given not by where we land on the number line, but by the amount we have jumped.

Using the empty number line for subtraction by "counting on"

This fourth method for subtraction is known as the "count-on" method, and it is based on the fact that it is easier to count forwards than backwards. As shown in the simple methods above, the "count-on" method for subtraction depends on finding the difference between two numbers by "counting on" from one to the other. This "counting on" can be shown by jumps along the empty number line. Let us return to our earlier example of

$$43 - 28 = 15$$

Using the "count-on" method we place both the 28 and the 43 on the empty number line and find the difference between them by jumping from the 28 to the 43



As in the simple example of $5 - 2 = 3$, when we use this method, our "answer", 15 is given by adding up the jumps along the number line. This differs from the other methods for both addition and subtraction in which the "answer" is given by the position in which we land on the number line after our final jump.

The first jump of 2 from 28 to 30 ensures that we are on a "comfortable resting place" which, as we have said before, can make large jumps easier.

In this section we have looked at how the empty number line can help with mental calculation, and be a very good way for children to record their mathematics.

- There are three empty number line methods that can be used for both addition and subtraction, we have called these the "Tens jumping" method, "Hit the tens" method and "Overjumping" method.
- There is another empty number line method that can be used for subtraction only. This is based on finding the difference between two numbers. We have called this method the "Count on" method.

Written methods for addition and subtraction

"Splitting" numbers to help with addition.

When we use the empty number line methods for addition we keep our first number whole and make a series of jumps to the value of the second number. There are other methods for addition which involve splitting both numbers up, and adding all the separate parts. These methods are based on the children's understanding of our number system. For example that the 4 in 45 stands for "four tens" which is 40 as we discussed in the **Mental methods** section on page 4.

We can use this splitting method to work out

$$37 + 29 = 66$$

by splitting both numbers into tens and ones, adding them separately then recombining like this:

$$37 = 30 + 7$$

$$29 = 20 + 9$$

$$30 + 20 = 50$$

$$7 + 9 = 16$$

$$50 + 16 = 66$$

This works because addition can be done in any order and we still get the same result:

$$2 + 5 = 7$$

and

$$5 + 2 = 7$$

Written methods for addition

We have discussed how written methods can involve lining the numbers up "vertically", one above the other and that these are called "column" methods because the digits line up in columns. Children need to be **very secure** with how our number system works, and what each digit in a number represents before they are able to use these methods without making mistakes.

Children will not be taught to use column methods for addition until they are confident with mental methods. The National Numeracy Strategy first introduces column methods at the end of Year 3, and they are developed from Year 4 onwards. This does not mean that all children will be using such methods in Year 3 and 4, Children in year 4 who are not confident with a mental method might not be asked to use column methods at all. They will use other methods such as number lines.

When children are first introduced to column methods it is really important that this builds on the mental methods that they are used to using.

Let us think about an addition: $36 + 57 = \square$

Children will be used to solving this type of addition mentally, perhaps by splitting the numbers into tens and ones.

36 is 3 tens and 6 ones 57 is 5 tens and 7 ones so

30 add 50 is 80 6 add 7 is 13

80 add 13 is 93 so $36 + 57 = 93$

The children are used to thinking of numbers in this way, and are used to adding the tens first.

It is important that when children are asked to start column addition, they can make use of the mental methods that they already know.

The children are shown that they can set out $36 + 57 =$ like this:

$$\begin{array}{r} 36 \\ + \underline{57} \end{array}$$

They are the asked to solve the problem like they are used to doing, by adding the tens first. They record their result underneath. The brackets remind them what they have done:

$$\begin{array}{r} 36 \\ + \underline{57} \\ 80 \quad (30 + 50) \end{array}$$

The next step is to add the ones and record the result underneath

$$\begin{array}{r} 36 \\ + \underline{57} \\ 80 \quad (30 + 50) \\ \underline{13} \quad (6 + 7) \end{array}$$

They can now add the 80 and the 13, which can be done easily mentally to give 93. This can be recorded underneath.

$$\begin{array}{r} 36 \\ + \underline{57} \\ 80 \quad (30 + 50) \\ \underline{13} \quad (6 + 7) \\ 93 \end{array}$$

Once children are used to this way of setting out the calculation, they are encouraged to try the addition by adding the ones first and then the tens. Do we get the same answer?

$$\begin{array}{r} 48 \\ + \underline{34} \\ 70 \quad (40 + 30) \\ \underline{12} \quad (8 + 4) \\ 82 \end{array} \qquad \begin{array}{r} 48 \\ + \underline{34} \\ 12 \quad (8 + 4) \\ \underline{70} \quad (40 + 30) \\ 82 \end{array}$$

Of course we get the same answer, but the children will need to discover this for themselves. Once confident with this, the children can extend this to using three digit numbers involving hundreds.

This approach is very different from suddenly asking children to add the ones first as soon as they start column methods, which would conflict with what we were asking them to do when they calculate mentally.

Let us try $235 + 367$

Adding the hundreds first

$$\begin{array}{r} 235 \\ + 367 \\ \hline 500 \text{ (200 + 300)} \\ 90 \text{ (30 + 60)} \\ \hline 12 \text{ (5 + 7)} \\ \hline 602 \end{array}$$

Adding the "ones" first

$$\begin{array}{r} 235 \\ + 367 \\ \hline 12 \text{ (5 + 7)} \\ 90 \text{ (30 + 60)} \\ \hline 500 \text{ (200 + 300)} \\ \hline 602 \end{array}$$

As children develop in their use of these methods, they will be taught to use them for addition of larger numbers, and numbers involving decimals. Column addition methods are particularly useful for adding more than two numbers involving decimals.

When children have had plenty of experience of this "expanded" method, and they can perform calculations accurately and with confidence, they may be introduced to the short cut, or "compact" method which involves "carrying". Some children may not move on to this stage.

When introduced to "carrying", the children will probably go back to two digit additions, like $37 + 28$

$$\begin{array}{r} 37 \\ + 28 \\ \hline 5 \\ \hline 1 \end{array}$$

First add the 7 and the 8 to give 15.

Because we understand our number system we know that this means one ten and five ones.

We can then place a 1 in the tens column to represent the one ten, and write the 5 in the ones column. We can then add the 30, 20 and the 10 to give 60. We represent this by writing a 6 in the tens column, six tens making 60.

$$\begin{array}{r} 37 \\ + 28 \\ \hline 65 \\ \hline 1 \end{array}$$

If children continue to make mistakes using this short cut, or "compact" method, they will go back to the "expanded" method.

Written methods for subtraction

We have seen how splitting numbers up can help with addition and that this works because addition can be done in any order and we still get the same result:

$$2 + 5 = 7 \quad \text{and} \quad 5 + 2 = 7$$

But this is not the case with subtraction:

$$5 - 2 = 3 \quad \text{and} \quad 2 - 5 = (-3) \text{ (negative 3)}$$

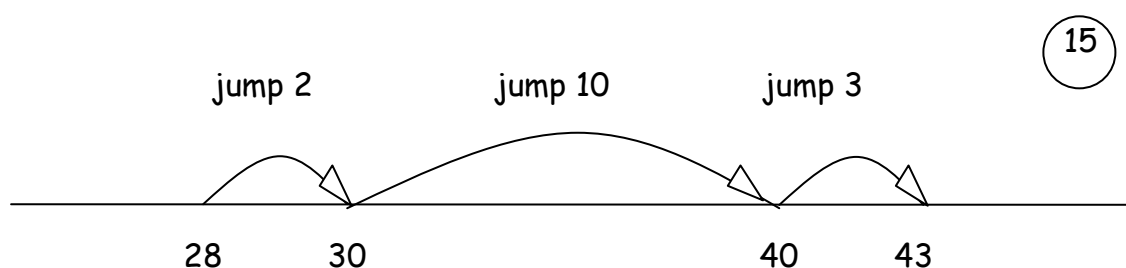
which is a very different result.

This means that splitting numbers up does not necessarily help when subtracting. We will now look at how the "count on" empty number line method for subtraction, which we first saw on **page 12** can be developed into a "column" method for subtraction.

Let us remind ourselves how we used this method to work out

$$43 - 28 = 15$$

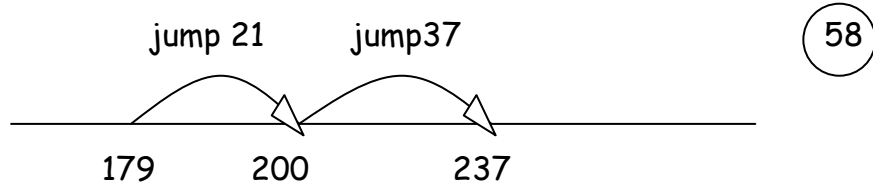
Using the "count-on" method we place both the 28 and the 43 on the empty number line and find the difference between them by jumping from the 28 to the 43



Our answer is given by adding up the jumps along the number line.

We will now use the same method, but this time the calculation involves three digit numbers: $237 - 179 = 58$

Place the 179 and the 237 on the number line and jump from one to the other to find the difference. Our answer is given by adding up the jumps.



We aim for a "comfortable resting place" on the number line. This could be 180 (our first "tens" number), but some children will be able to go straight for 200. Our answer is given by adding our jumps, so it is in our interest to try to reduce the number of jumps to make our calculation more efficient.

As children develop their use of this method, they will probably need to jot down their jumps to keep track of the calculation, although the actual **work** of the calculation is done in their heads. Once children are confident with this as a mental method helped by the empty number line, the same mental method can be used, but this time supported by a column recording method.

We will use this column method to solve $237 - 179$

$$\begin{array}{r} 237 \\ - 179 \\ \hline 21 \text{ (200)} \\ 37 \text{ (237)} \\ \hline 58 \end{array}$$

Set the numbers out one above the other, with the digits lined up. We then jump on in easy steps, first from 179 to 200, which is a jump of 21. We can record where we have got to in brackets at the side. We then jump from 200 to 237 which is a jump of 37. Add up our jumps to give 58.

This column method is just a different way of recording the jumps, as can be seen by comparing the same calculation using the empty number line "count-on" method.

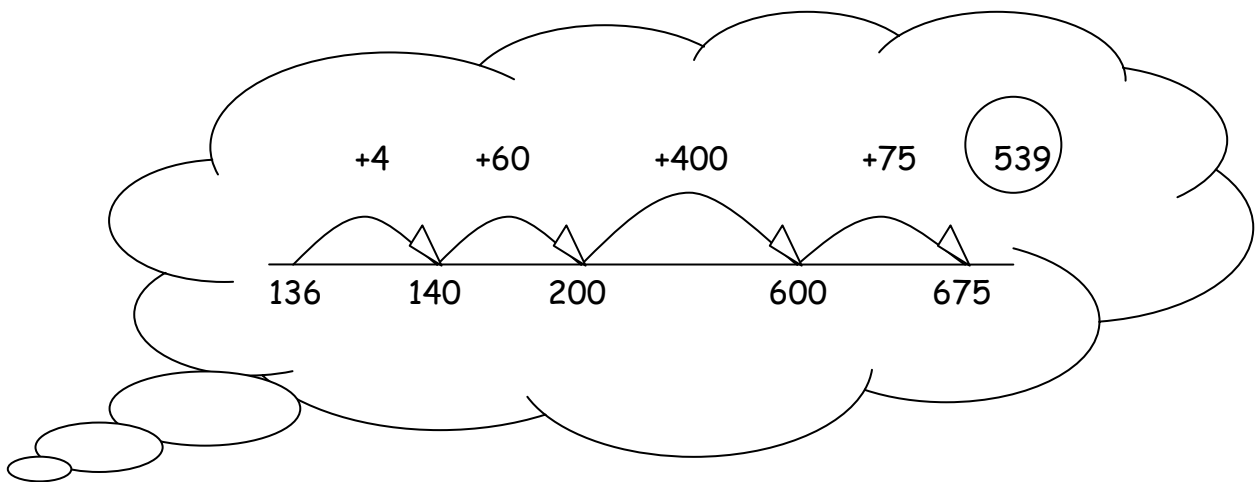
Whilst using this method the children can be taught to use the number line as a mental image for their thinking. If children continue to make mistakes using the column method, they might be asked to go back to the number line to record their thinking.

We will now use this column method to solve $675 - 136$

$$\begin{array}{r} 675 \\ -136 \\ \hline 4 \quad (140) \\ 60 \quad (200) \\ 400 \quad (600) \\ \hline 75 \quad (675) \\ \hline 539 \end{array}$$

Our first jump will be from 136 to 140 which is 4. Next jump from 140 to 200 which is 60. Next jump from 200 to 600 which is 400. Final jump from 600 to 675 which is 75. Add up all the jumps to give 539.

Whilst doing this calculation it can help to have the number line in our minds as a mental image:



This method of subtraction, using the number line or the "column" method can be extended to larger numbers and decimals. You might like to try some!

Methods for Multiplication and Division

Understanding multiplication

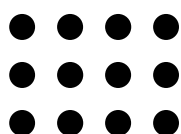
When children first learn about multiplication, they are taught that multiplication is repeated addition.

For example:

4 added together 3 times is : $4 + 4 + 4$

This can be thought of as 3 lots of 4, or four, three times (this is usually expressed as 3 times 4) and that we can write this as 4×3

Arranging numbers in a grid (sometimes called an array) can be a useful way of understanding multiplication. This grid shows 3 rows of 4



Looking at the rows of 4 dots, this can be thought of as 3 lots of 4, or $4 + 4 + 4$ or 4×3

Looking at each column of 3 dots, the grid can be thought of as 4 lots of 3, or $3 + 3 + 3 + 3$ or 3×4

So there are many different ways of describing this arrangement of dots, but the result is always the same.

So 4×3 and 3×4 give the same result of 12

If we know that $3 \times 4 = 12$ then we also know that $4 \times 3 = 12$

Learning multiplication facts (Tables)

Once children understand that multiplication means repeated addition and can use this knowledge to solve simple problems, they are faced with the challenge of starting to learn by heart the multiplication facts up to 10×10 .

Children begin this process by learning to count in various numbers, starting with twos and tens. They will become familiar with the two "times table" through learning the doubles of numbers, and will then need to be able to answer questions such as "what are six tens?" straight away.

The next stage in the process is to learn by heart the 5 "times table" and begin to learn the 3 and 4 "times tables"

The final stage is to learn the 6,7,8 and 9 "times tables"

The National Numeracy Strategy sets out expectations for when children should know these tables by heart. This is a gradual but steady process starting in Year 2 and the expectation is that children know all the multiplication facts up to 10×10 by the end of Year 5.

It is worth noting that "knowing by heart" means being able to answer a question such as $6 \times 7 = \square$ straight away, without having to count up through all of the table first. It is important that children use the fact that 6×7 gives the same result as 7×6 , so if we know one, we know the other.

Parents and carers can play an important role supporting schools in helping children to learn the multiplication facts. Providing time at home, helping children to practise, motivating and encouraging children and above all praising success are vitally important. There are computer based activities, books, musical tables and other games available, all of which might help to motivate children, but learning all of these facts is hard work, and they will need plenty of encouragement from school and home. Without quick recall of multiplication facts, children will find it difficult to use written methods for more complicated multiplications.

To summarise:

- Children need to learn their "tables", and this is a steady process from Year 2 to Year 5 and beyond.
- This is hard work, and children will need support and encouragement.
- If you are unsure as to how to support your child in learning their "tables" the best advice is to contact your child's school.

Multiplying and dividing by 10

Because our number system is based on 10, multiplying and dividing by 10 is easy, but it is important that children understand what is going on. By performing simple multiplications with a calculator, children will soon notice a pattern such as:

$$\begin{aligned}8 \times 10 &= 80 \\6 \times 10 &= 60 \\12 \times 10 &= 120\end{aligned}$$

When asked to describe this pattern it is tempting to say something like "when you multiply by ten you put a zero on the end of the number". This might achieve correct answers for a while, but it does not tell the whole story, and is certainly not useful when we start to multiply decimals by ten for example $1.2 \times 10 = 12$

A useful model is to consider the result of multiplying by 10 on numbers in a column chart.

H	T	U
		6
	6	0

By putting the numbers in a chart such as this one we can see that multiplying 6 by 10 results in the 6 moving one place to the left, to the "tens" column. A zero needs to be placed in the "units" or "ones" column to show that we are now dealing with 6 tens.

Conversely, if we want to divide 80 by 10, the 80 will move one place to the right to give 8. This can be extended to multiplying and dividing by 10 and 100, and the chart can easily be extended in both directions to enable any number to be multiplied or divided by 10 or 100, or indeed any multiple of 10.

H	T	U	. 1/10
	8	0	
		8	
	1	6	.
		1	. 6

This chart has been extended to show the decimal point and the "tenths" column. To solve $16 \div 10$ we place 16 in the chart, and move it one place to the right to give 1.6

When children know the multiplication facts up to 10×10 , and also understand how to multiply by ten, they can combine these two pieces of knowledge and multiply by multiples of ten for example:

$$7 \times 30 = \square$$

We know that 30 is ten lots of 3, so to work out 7×30 we solve 7×3 and multiply this by 10.

We know by heart that $7 \times 3 = 21$

we also know that $21 \times 10 = 210$

so we have worked out that $7 \times 30 = 210$

Mental and written methods for multiplication.

As with addition and subtraction there is a wide range of mental methods for multiplication. Some examples are:

Children will use their knowledge of doubles to multiply by 2, and later to multiply by 4, and 8 by doubling and doubling again. Knowledge of doubling and halving can be used to multiply by 5, by multiplying by 10 and halving the result.

Multiplication by 9 or 11 can be performed by first multiplying by 10, then adding or subtracting, for example:

$$\begin{aligned}17 \times 11 &= (17 \times 10) + 17 \\ &= 170 + 17 \\ &= 187\end{aligned}$$

$$\begin{aligned}17 \times 9 &= (17 \times 10) - 17 \\ &= 170 - 17 \\ &= 153\end{aligned}$$

Once again children are encouraged to explain their methods, compare their own with those of other children and the teacher, and see how particular methods are suited to certain types of calculation.

"Splitting" numbers to help with multiplication

In a similar way to addition, multiplication can be done in any order and we have already seen how 6×7 and 7×6 give the same answer. This fact allows us to split one or both numbers when multiplying, in the same way as we did in preparation for written methods for addition.

In this example we know that 37 can be split into 30 and 7
We can multiply each of these numbers by 6, and combine the results.
We can work out 30×6 mentally because we know by heart that $3 \times 6 = 18$ and we know that $18 \times 10 = 180$

we know by heart that $7 \times 6 = 42$

we then add $180 + 42 = 222$

This can be written like this:

$$\begin{aligned}37 \times 6 &= (30 \times 6) + (7 \times 6) \\ &= 180 + 42 \\ &= 222\end{aligned}$$

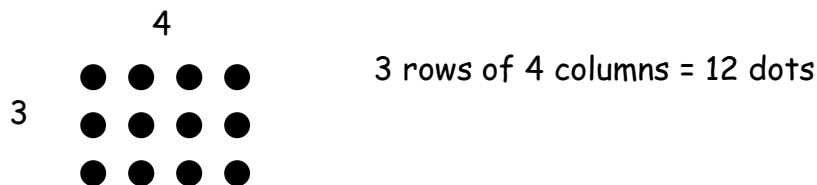
This works in every case, and it is a useful method to think about as we move towards written methods for multiplications that are too difficult to calculate mentally.

Written methods for multiplication

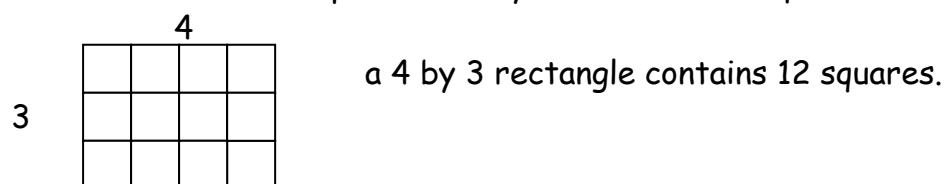
The grid method.

The grid method for multiplication, as its name suggests is a written method which is based on a grid or "array" (see page 21)

Remember how we could represent $4 \times 3 = 12$



4×3 could also be represented by the number of squares in a rectangle:



This, combined with the method of splitting numbers to help with multiplication, gives us an effective written method for multiplication.

Let us again think about 37×6

We split the 37 into 30 and 7, but this time we use a grid to help us record the calculation. We put the 30 and 7 along the top of the grid, and the 6 we are multiplying by down the side like this:

x	30	7	
6	180	42	= 222

We can now work out the multiplication for each rectangle in the grid separately:

We can work out 30×6 from our knowledge that $3 \times 6 = 18$ so $30 \times 6 = 180$, this is written in the appropriate rectangle.

We know that $6 \times 7 = 42$ and this again is recorded in the appropriate rectangle.

We then add the numbers in the two rectangles

$$180 + 42 = 222$$

so we have solved

$$37 \times 6 = 222$$

This method can be used for larger numbers simply by increasing the number of rectangles in the grid.

To calculate 357×8 we need a grid with three rectangles. We then split the 357 into 300, 50, and 7.

We place these numbers, together with the 8 that we are multiplying by, around the grid.

x	300	50	7
8			

As before we work out the multiplication for each rectangle separately, and we can do this in any order. We will start with the largest number:

300×8 We can work this out because we know that $3 \times 8 = 24$ and we now need to multiply 24 by 100.
We can do this simply from our knowledge of multiplying by ten, or multiples of ten.

$24 \times 100 = 2400$ We write this in the correct rectangle.

x	300	50	7
8	2400		

We now work out 50×8 in a similar way.

50×8 We know $5 \times 8 = 40$ and multiply this by ten

$40 \times 10 = 400$ We write this in the correct rectangle.

x	300	50	7
8	2400	400	

The final multiplication in this example is $7 \times 8 = 56$ which once again we record in the correct rectangle

x	300	50	7
8	2400	400	56

We now need to add the numbers in the rectangles to complete the calculation: $2400 + 400 + 56 = 2856$

We can use any method of our choice to add these numbers, in this example the addition is fairly straightforward and can be done mentally.

So far we have used the grid method for multiplying by a single digit number, but the method can easily be used for multiplying by larger numbers simply by adding an extra row to the grid. A two digit by two digit multiplication can be performed by using a two by two grid, and splitting both numbers as before:

This example shows a completed grid for working out 73×54

x	70	3	
50	3500	150	3650
4	280	12	<u>292</u>
			3942

The top left rectangle contains the result of 70×50 which we solve, as before from our knowledge that $7 \times 5 = 35$ and our knowledge of multiplying by 10 or 100. Once all the rectangles are filled in, the numbers in each are added, the total giving us the result of the multiplication.

This grid method can be extended in a similar way to perform three digit by two digit multiplication and beyond, by splitting both numbers.

Here is a grid ready for calculating 243×36

x	200	40	3
30			
6			

The grid method can also be applied to multiplications involving decimals. We will now use the grid method to solve 34.5×32

We need a three by two grid, and split both numbers.

x	30	4	0.5	
30	900	120	15	1035
2	60	8	1	<u>69</u> 1104

To solve 30×0.5 we could use our knowledge that $3 \times 5 = 15$, and our knowledge of dividing by 10, or we could use the fact that 0.5 is equivalent to one half, and that 15 is one half of 30.

The grid method is an efficient written method for multiplication which builds upon the children's mental methods, and provides an effective mental image for multiplication. This method is used successfully by children, and might be their preferred method for multiplication right through to Year 6 and beyond.

Column methods for multiplication

We have seen how the grid method for multiplication provides us with a logical written method for "long multiplication" which builds on the mental methods that children use. As we have said, the grid method might be the preferred method for multiplication for many children, being one that they understand, can use quickly and obtain correct answers.

There are column methods for multiplication which involve lining numbers up one above the other in a more traditional way. There is no advantage to children using column methods if they lead to errors, but the column method we will look at here uses exactly the same mathematics as the grid method, but is written down in a different way.

Let us look again at the relatively simple example of 37×6 recorded here in the grid method.

$$\begin{array}{r} \times \quad 30 \quad 7 \\ 6 \quad \boxed{180} \quad \boxed{42} \quad = \quad 222 \end{array}$$

It is very clear from the grid layout that we have to perform 2 multiplications, (30×6 and 7×6) and add the results. When children are first learning column methods (as with addition) it is useful to think of them as "column methods for recording". In this case we will do exactly the same mathematics as for the grid method, but record it in a different way, in columns:

$$\begin{array}{r} 37 \\ \times \quad 6 \\ \hline 30 \times 6 \quad 180 \end{array}$$

We will start by multiplying 30×6 to give 180. We record this below the line. The multiplication written beside the column help to remind us what we have done

$$\begin{array}{r} 37 \\ \times \quad 6 \\ \hline 30 \times 6 \quad 180 \\ 7 \times 6 \quad 42 \\ \hline 222 \end{array}$$

The next step is to multiply 7×6 and record this below the line. Once again the multiplications written beside the column helps to remind us what we have done. We complete the calculation by adding the 180 and 42 to give 222.

It is important to note that, as with addition children are used to dealing with the tens first.

It is useful to make the link between the two multiplication "steps below the line" in the column recording method, and the two rectangles which need to be filled in when using the grid method.

We will now return to a two by two digit multiplication, using the grid method:

$$73 \times 54$$

	70	3	
50	3500	150	3650
4	280	12	<u>292</u>
			3942

Now we see the same calculation using the column method. There are 4 rectangles in the grid, which correspond to the 4 multiplication calculations necessary to solve the problem. When using the column recording method, we find that there are 4 corresponding multiplication "steps below the line" which are then added:

$$\begin{array}{r}
 73 \\
 \times \underline{54} \\
 70 \times 50 \quad 3500 \\
 3 \times 50 \quad 150 \\
 70 \times 4 \quad 280 \\
 3 \times 4 \quad \underline{12} \\
 \hline
 3942
 \end{array}$$

As children gain experience with the grid method, and the corresponding column method, in the same way as we saw for addition it becomes obvious that it does not matter in which order the different "steps" or components of the multiplication are completed. They always add up to give the correct result. With this knowledge it is then possible to move towards the more traditional column method which requires multiplying by the units or ones first, and "carrying".

To illustrate this we will look again at 37×6

$$\begin{array}{r} 7 \\ \times 6 \\ \hline 7 \times 6 \\ 30 \times 6 \\ \hline 222 \end{array}$$

In this example the calculation has been performed by multiplying 7×6 to give 42 then multiplying 30×6 to give 180

$$\begin{array}{r} 7 \\ \times 6 \\ \hline 222 \\ 4 \\ \hline \end{array}$$

This can lead to the short cut method, in which the calculation 7×6 is performed and recorded as 42 with the 4 "carried" to the tens column. This method requires the children to be very secure with the number system.

It is worth remembering that the aim is for children to have written methods that they can use reliably and accurately, and that there is no advantage to children using short cut column methods unless they understand the mathematics behind the method.

Understanding division

Sharing and grouping

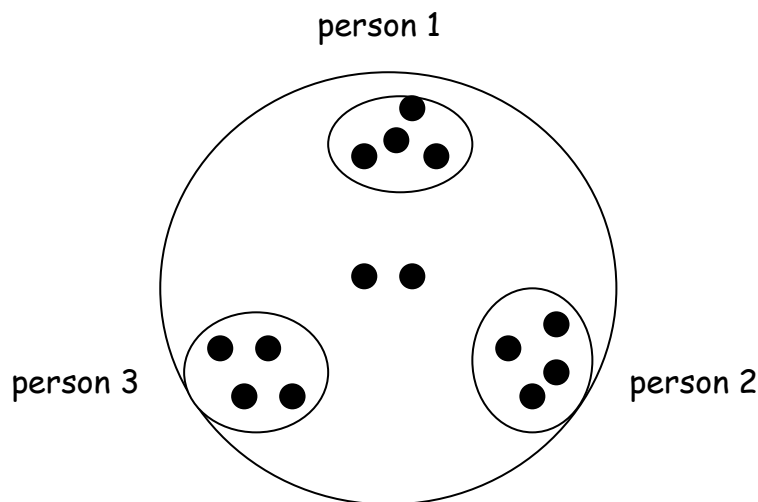
Children are taught to think about division in two ways, as **sharing** and as **grouping**. Both sharing and grouping are examples of division, and both can be done as practical activities. It is worth trying the following three activities with a small packet of sweets at home.

Here is a simple division

$$14 \div 3 =$$

We will first think about this as sharing. We have 14 sweets to share equally between 3 people. Each person has 4 sweets with 2 left over.

If we demonstrate this by sharing sweets, one by one around the table, the results will look like this:



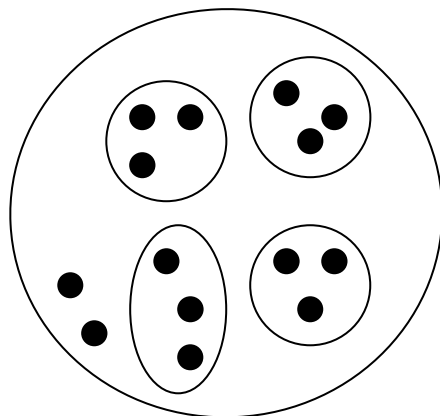
The answer to our problem is:

$$14 \div 3 = 4 \text{ remainder } 2$$

This is given by the number of sweets held by each person, and the remainder is the number of sweets left because they could not be shared equally without breaking them up into parts.

We can shorten this to $14 \div 3 = 4 \text{ r } 2$

We will now think the same division calculation but this time we will not be sharing the sweets, but putting them into groups, or grouping. We still have 14 sweets, but we want to put the sweets into groups of 3. How many groups will there be?



This diagram shows the sweets grouped into groups of 3. The answer is given in this case, not by the number of sweets in each group, but by the number of groups of three that we can make.

So $14 \div 3 = 4 \text{ r}2$ can be represented by grouping as

14 sweets arranged in groups of 3, there are 4 groups with 2 sweets left over because they cannot be made up in to a further group of 3.

We can take the idea of grouping one step further, and make a link between division and subtraction. In the same way that children learn about multiplication as repeated addition (page 21), division can be thought of as repeated subtraction.

Let us think about $14 \div 3$

We can think of this as "How many times can I subtract 3 from 14?"

We can take one group away, there are 11 left.

Take a second group away, there are 8 left.

Take a third group away, there are 5 left.

Take a fourth group away, there are two left

We cannot take any more groups of 3 away because there are only 2 left.

So $14 \div 3$ gives us 4 groups, 2 left over. $14 \div 3 = 4 \text{ r}2$

The written methods for division that we will look at later are based on understanding division as repeated subtraction.

Mental and written methods for division

Mental methods

We have already seen the importance of multiplying and dividing by 10 and 100 (page 23).

Children will use their knowledge of halving to divide by 2, and later extend this to halving and halving again to divide by 4.

When learning multiplication facts (see page 22) it is important to extend our understanding of multiplication to make the link between multiplication and division,

if we know that $4 \times 6 = 24$
we also know that: $6 \times 4 = 24$
and that $24 \div 6 = 4$
and that $24 \div 4 = 6$

Written methods for division

The written method for division that we will look at here is based on understanding division as repeated subtraction, and this method is sometimes called "chunking".

Before looking in detail at the method it is worth making clear that this symbol is often used for division:

)

and that:

$6 \overline{) 28}$ means the same as $28 \div 6 =$

and that whichever symbol is used, the calculation can be worked out in the same way.

The "chunking" method for division

Let us look at the example $148 \div 6$

We know from our understanding of division as grouping and repeated subtraction that we can solve this example by saying "how many groups of 6 can I subtract from 148?"

So we could work this out as:

$$\begin{array}{r} 148 \\ - \quad 6 \\ \hline 142 \\ - \quad 6 \\ \hline 136 \\ - \quad 6 \\ \hline 130 \\ \downarrow \end{array}$$

...and so on until we are left with less than 6. We could then count up the number of times we subtract the 6. This method would work but it would take a long time and there is a high likelihood of errors.

We can do the same thing, but make it much more efficient by subtracting more than one group of 6 at a time. An easy number of groups to start with is 10 groups of 6.

$$\begin{array}{r} 148 \\ - \quad 60 \quad (6 \times 10) \\ \hline 88 \\ - \quad 60 \quad (6 \times 10) \\ \hline 28 \\ - \quad 12 \quad (6 \times 2) \\ \hline 16 \\ - \quad 12 \quad (6 \times 2) \\ \hline 4 \end{array}$$

We start by subtracting ten lots of 6 from 148. $148 - 60 = 88$
We can then subtract another ten lots of 6 from 88. $88 - 60 = 28$
We cannot subtract a further ten lots of 6 without going below zero, so our next step is to decide how many lots of 6 to subtract. We can try 2 lots of 6. $28 - 12 = 16$. Now subtract another two lots of 6. $16 - 12 = 4$.
We are left with 4

We can use the numbers in the brackets to add up how many lots of 6 we subtracted from 148. We subtracted a total of 24 lots of 6, and were left with 4.

We have solved $148 \div 6 = 24 \text{ r } 4$

As children practise the "chunking" method, they are taught to make the calculation more efficient by decreasing the number of steps, by using their knowledge of mental methods.

We will look again at $148 \div 6$ and this time try to make the calculation more efficient.

We know that $6 \times 2 = 12$, and from our knowledge of multiplying by ten we know that $6 \times 20 = 120$

120 is not too far away from 148 so we will start our calculation by subtracting 20 lots of 6 from 148

$$\begin{array}{r} 148 \\ - \underline{120} \quad (6 \times 20) \\ 28 \\ - \underline{24} \quad (6 \times 4) \\ 4 \end{array}$$

We are left with 28.
We know that $6 \times 4 = 24$,
so we will now subtract
four lots of 6, leaving us
with 4

Once again we add the numbers in brackets to add up how many lots of 6 we have subtracted from 148.

The chunking method can be set out using this symbol for division:

$$6 \overline{) 148}$$

but the calculation is done in exactly the same way, by subtracting lots of 6 from 148

$$\begin{array}{r} 6 \overline{) 148} \\ - \underline{120} \quad (6 \times 20) \\ 28 \\ - \underline{24} \quad (6 \times 4) \\ 4 \end{array}$$

Answer 24 r 4

This method can be applied to more difficult examples including decimals.
We will use the method to work out:

$$258 \div 17$$

$$\begin{array}{r} 258 \\ - 170 \quad (17 \times 10) \\ \hline 88 \\ - 85 \quad (17 \times 5) \\ \hline 3 \end{array}$$

Answer 15 r 3

Understanding remainders after division

An important aspect of division is understanding what to do with the remainder, particularly when we are using division to solve problems.

For example, this problem:

"Fourteen children go camping. The tents they use can sleep a maximum of three children. How many tents will they need to take?"

can be solved by the division $14 \div 3 = 4$ remainder 2

But there is no such thing as "4 remainder 2 tents", we need to round up our number of tents to 5 so that there are enough tents for everyone to sleep in.

Another problem:

A farmer has 32 eggs to pack in boxes of 6. How many egg boxes will be full?

$$32 \div 6 = 5 \text{ remainder } 2$$

For this problem we need to understand that there will be 5 full boxes and 2 eggs left over.

Problem solving and calculations

We have looked in this booklet at mental and written calculation methods for addition, subtraction, multiplication and division. As children learn methods for each of these, they are taught to apply their methods to problems involving stories, money and measurement.

When given problems to solve, children tend to use the methods that they are most confident with, which are those methods that they understand thoroughly.

When faced with a calculation to work out, children are taught to choose the most appropriate method by asking themselves:

- Can I do this in my head?
- If I can't do it in my head, what do I need to write down that could help me calculate the answer?
- Will the written method I know be helpful?
- Is it sensible to use a calculator?
- Is my answer sensible?

Mathematical activities at home

Mathematics is not all about hard calculations! Maths is all around us, and the last section of this booklet contains some suggestions for parents and carers to make the most of the mathematical opportunities in everyday life.

Playing games

- At an early stage young children learn to recognise that a number can stand for a set of objects in a group, for example the five spots on a domino can be matched to the number 5.
- All board games involving moving around a board can give experience of counting on and landing on a new number.
"How many spaces can you move?"
"How many more do you need to win?"
- Games involve learning to take turns, deal out the right number of cards or pieces and co-operating in using the equipment.
"What are the chances of rolling a six?"
"Is it harder to roll a six than any other number?"
- Games involving keeping a score provide excellent opportunities for calculating how far ahead or behind you are, and how many points you need to win.
"How can you throw three darts to finish from a score of 110?"

Using the computer

- There are various pieces of computer software which are designed provide exciting and enjoyable ways of practising mathematics, learning tables etc.
- Children can use data handling packages to produce graphs from surveys done at home.
"Export the graph into a word processing package and write about what the graph tells you."
"Make up some questions that can be answered using the graph."

Shopping

- Younger children can handle and count out coins in simple transactions, and as they gain more experience can work out the change required.
"How much will these cost?"
"How much money do we need?"
"How much change will we get?"
- Older children can use a calculator keep a record of supermarket shopping.
"Is the bill correct?"
"If we spent this much each week, how much will we spend in a year?"

Measuring

All types of measuring give real meaning to using numbers.

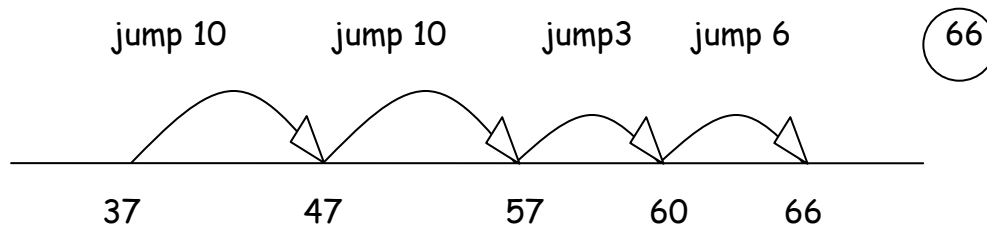
- Following a recipe and weighing out amounts accurately will give valuable practice in reading scales.
- On long car journeys the question "are we nearly there yet?" can be answered by discussing whether we mean in time or distance.
"Follow the route on a map and estimate how much more time the journey will take."
"How far do you think we have travelled?"
"Use the trip meter to check the distance"
"Use a calculator to work out our average speed"
- When telling the time children are more used to digital clocks than conventional clock faces. If possible have both in the house and compare the two. Talk about what time different things happen during the day.
"When does your favourite TV programme start?"
"How long does it last?"
"How much time is left on the videotape?"
"How many weeks is it until your birthday?"
"How many days is this?"

Quick reference: Methods shown in this booklet

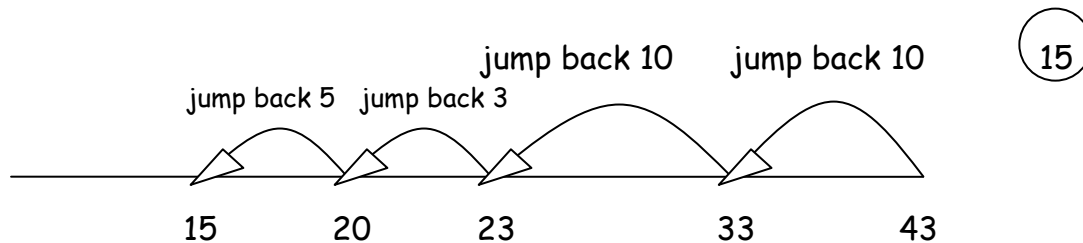
Empty number line methods for addition and subtraction

The "Tens jumping" method

Addition $37 + 29 = 66$

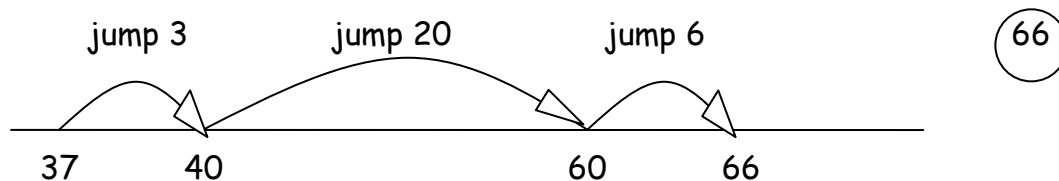


Subtraction $43 - 28 = 15$

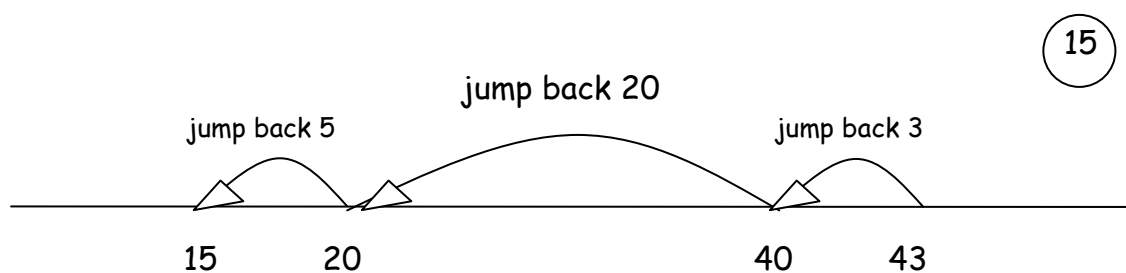


The "Hit the tens" method

Addition $37 + 29 = 66$

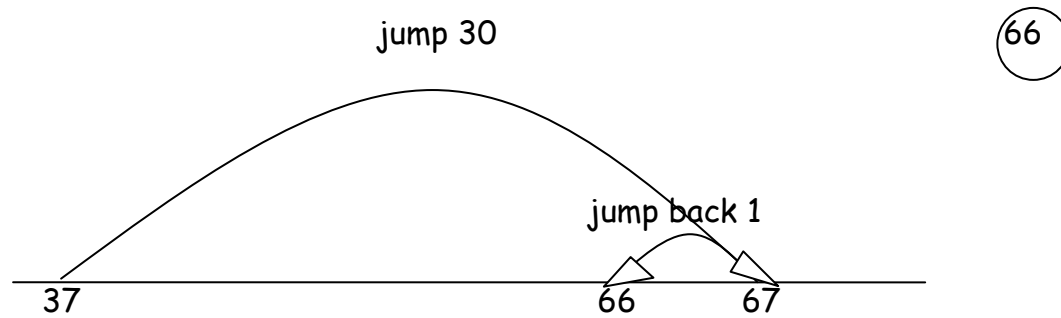


Subtraction $43 - 28 = 15$

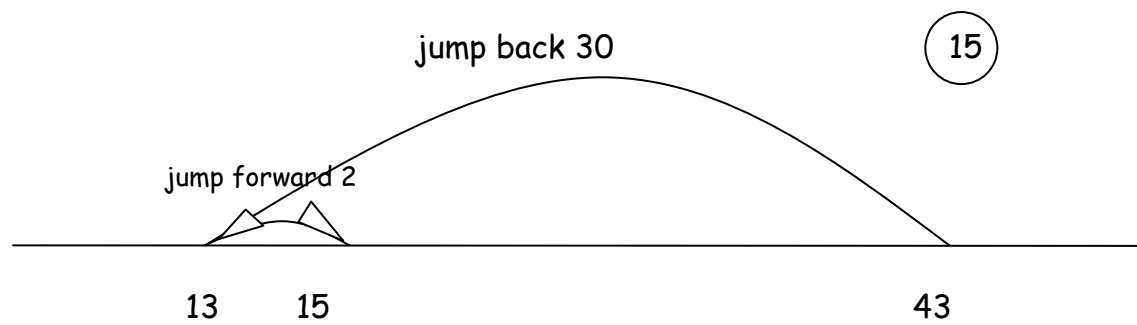


The "Overjumping" method

Addition $37 + 29 = 66$



Subtraction $43 - 28 = 15$



Written methods for addition:

Column addition

$$235 + 377$$

Adding the hundreds first

$$\begin{array}{r} 235 \\ + \underline{367} \\ \hline 500 \text{ (200 + 300)} \\ 90 \text{ (30 + 60)} \\ \underline{12} \text{ (5 + 7)} \\ \hline 602 \end{array}$$

Adding the "ones" first

$$\begin{array}{r} 235 \\ + \underline{367} \\ \hline 12 \text{ (5 + 7)} \\ 90 \text{ (30 + 60)} \\ \underline{500} \text{ (200 + 300)} \\ \hline 602 \end{array}$$

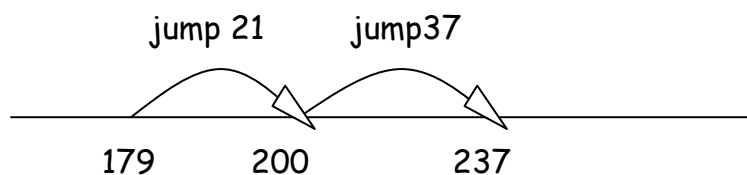
Short cut method with "carrying"

$$\begin{array}{r} 235 \\ + \underline{367} \\ \hline 602 \\ \small 11 \end{array}$$

Written methods for subtraction:

The "Count-on" method using the empty number line

$$237 - 179$$



Using the column recording method

$$\begin{array}{r} 237 \\ - \underline{179} \\ \hline 21 \text{ (200)} \\ \underline{37} \text{ (237)} \\ \hline 58 \end{array}$$

Written methods for multiplication

The grid method

$$73 \times 54$$

	70	3	
50	3500	150	3650
4	280	12	<u>292</u>
			3942

Column methods for multiplication

$$73 \times 54$$

$$\begin{array}{r} 73 \\ \times 54 \\ \hline 70 \times 50 \quad 3500 \\ 3 \times 50 \quad 150 \\ 70 \times 4 \quad 280 \\ 3 \times 4 \quad \underline{12} \\ \hline 3942 \end{array}$$

The "chunking" method for division

$$\begin{array}{r} 6 \overline{) 148} \\ - \underline{120} \quad (6 \times 20) \\ 28 \\ - \underline{24} \quad (6 \times 4) \\ 4 \end{array}$$

Answer 24 r 4